Lesson 5. Fixed Points of First-Order Linear DS, Discrete Market Models

0 Warm up

Example 1. Consider the first-order linear DS $A_{n+1} = sA_n + b$, $n = 0, 1, 2, \ldots$. Assume $s \neq 1$. Find the fixed points.

1 Fixed points of first-order linear DS

1.1 When $s \neq 1$

- Consider the first-order linear DS $A_{n+1} = sA_n + b$, $n = 0, 1, 2, \ldots$
- We found the fixed point of this DS in Example 1 when $s \neq 1$:
- Is this fixed point attracting or repelling?
- Recall the general solution to this DS is
- If $|s| < 1$, then

  which means the fixed point is
- If $|s| > 1$, then

  which means the fixed point is

Example 2. Consider the DS $A_{n+1} = -A_n + b$, $n = 0, 1, 2, \dots$. Write the first few terms of A_n.

- Example 2 shows us what happens when $s = -1$: the fixed point is

1.2 When $s = 1$

- If $s = 1$ and $b \neq 0$, then

- If $s = 1$ and $b = 0$, then

2 Discrete market models

- A **discrete market model** describes the evolution of prices, supply, and demand of some product at discrete time points

- Variables:

- Equations:

- In other words:
  - The supply at time is determined by the price at time
  - The demand at time is determined by the price at time
- We can convert the equations above into a first-order linear DS describing the price of the product with some algebraic manipulation:

- The general solution to this DS is

- The single fixed point is
- Therefore, we can rewrite the general solution as

- Notice that $P_t \to \bar{P}$ when $(-\frac{d}{b})^t \to 0$, which only happens when

Example 3. In the discrete market model, suppose $P_0 = \bar{P}$. What does P_t equal for all t? Why does your answer make sense?